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Black hole thermalization rate from brane anti-brane model

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Abstract: We develop the quasi-particle picture for Schwarzhchild and far from extremal black holes. We show that the thermalization equations of the black hole is recovered from the model of branes and anti-branes. This can also be viewed as a field theory explanation of the relationship between area and entropy for these black holes. As a by product the annihilation rate of branes and anti-branes is computed.

Keywords: D-branes, Black Holes in String Theory, Tachyon Condensation.

Contents

1. Introduction

The properties of certain near extremal black holes are encoded in the thermodynamics of large-N gauge theories [1, 2]. Recently it was shown that the properties of the large-N gauge theories at finite temperature in the large 't Hooft effective coupling regime can be captured by a simple quasi-particle description [3, 4]. In this description each quasi-particle has an energy of order the temperature and thus the number of quasi-particles is the entropy of the gauge theory. Further the lifetime of the quasi-particles is of order the inverse temperature. For the case where only one type of brane is considered, the horizon size follows from the double peaked nature of the spectral density of the scalars fields [5].

Apart from describing the equilibrium properties of the system the quasi-particle description captures the approach to equilibrium too. It was shown that in general the membrane paradigm description of a black hole gives the equation [3]

$$\frac{dM}{dt} \sim \frac{A}{G} T^2 \tag{1.1}$$

describing the response to perturbation and the approach to equilibrium of the black hole. The quasi-particle description of the near extremal black holes gave

$$\frac{dE}{dt} \sim N_{qp} T^2 \tag{1.2}$$

where N_{qp} is the number of quasi-particles. This lead to the identification (in these cases $dE = dM$)of the number of quasi particles with the area of the horizon in Planck units, giving a simple explanation of why the entropy is proportional to the area in Planck units. This approach was generalized to a variety of examples including the rotating BTZ black hole.

More recently following the work in [6], it has been shown that a large class of black hole (far from extremal) [7]–[13] can be described using near extremal black holes, with opposite charges. Since the quasi-particle description gave a simple explanation to some of the properties of the near extremal black hole it seems that maybe the quasi-particle picture can explain some properties of the far from extremal black hole (including Schwarzhchild black holes). In this note we start to develop this quasi-particle picture.

2. Single charged black brane

We first want to look at the case where there are some brane and some anti-branes of the same type. The black brane properties are described by [14, 15]

$$\begin{aligned} M_{bh} &= \frac{\omega_{d+1}}{2\kappa^2} L^p \mu^d (d+1+d\sinh^2\gamma) \\ S_{bh} &= \frac{2\pi}{\kappa^2}\omega_{d+1} L^p \mu^{d+1}\cosh\gamma \\ Q_{bh} &= \frac{1}{2\sqrt{2}\kappa}\omega_{d+1} L^p \mu^d \sinh 2\gamma \\ T_{BH} &= \frac{d}{4\pi\mu cosh\gamma}\,. \end{aligned} \tag{2.1}$$

In [11] these black holes are modeled by the properties of the brane anti-brane system at finite temperature. The field theory on the brane is described by the thermodynamical relation

$$\begin{aligned} S &= aE^\lambda\sqrt{M_{p,0}} \\ \lambda &= \frac{d+1}{d} - \frac{N}{2} \quad D = p+d+3 \\ a &= 2^{\frac{1}{d}+2}\pi\omega_{d+1}^{-\frac{1}{d}} d^{-\frac{d+1}{d}}\lambda^{-\lambda} L^{-\frac{p}{d}}\kappa^{\frac{2}{d}} \\ M_{p,0} &= L^p\tau_p N \end{aligned} \tag{2.2}$$

where τ_p is the tension of the brane , N is the number of branes and E is the energy of the excitations. Similarly the theory on the anti brane is described by the same relationship with $M_{p,0}$ replaced by $M_{\bar{p},0}$, and E replaced by $\bar{E}$. It was argued that for stability $E=\bar{E}$, and we will continue to assume this. Maximizing the total entropy of the brane and anti-branes subject to this constraint and at a fixed total mass and charge gave the relationship

$$E_{\text{total}} = 2E = 4\lambda\sqrt{M_{p,0}M_{\bar{p},0}}. \tag{2.3}$$

If we define μ and γ such that they are related to the mass and charge of the black hole as in (2.1) then we find [11]

$$\begin{aligned} M_{p,0} &= \frac{\omega_{d+1}}{2\kappa^2} L^p \mu^d \frac{d}{4} e^{2\gamma} \\ M_{\bar{p},0} &= \frac{\omega_{d+1}}{2\kappa^2} L^p \mu^d \frac{d}{4} e^{-2\gamma} \\ E_{\text{tot}} &= \lambda\frac{\omega_{d+1}}{2\kappa^2} L^p \mu^d d\,. \end{aligned} \tag{2.4}$$

The theories while having the same energy have different temperatures, T and $\bar{T}$ respectively, satisfying

$$\frac{2}{T_{BH}} = \frac{1}{T} + \frac{1}{\bar{T}}\,. \tag{2.5}$$

Following the analysis in [3], the rate at which the black hole emits energy as seen by observers at the stretched horizon, ignoring charged emission (So from now on the charge Q is fixed), is given by,

$$\frac{dM_{BH}}{dt} \sim \frac{A}{G} T_{BH}^2 \sim S_{BH} T_{BH}^2\,. \tag{2.6}$$

If we are also interested in perturbation that keep the charge constant, then $dM = T_{BH} dS_{BH}$, and one has

$$\frac{dS_{BH}}{dt} \sim \frac{A}{G} T_{BH} = S_{BH} T_{BH} \,. \tag{2.7}$$

Equations (2.6) and (2.7) describe how the black hole thermalizes.

Since we claimed that the black hole described in equation (2.1), can be modeled by field theory living on branes and anti-branes we should be able to reproduce equations (2.6) and (2.7), using the out of equilibrium properties of the near extremal branes.

According to the quasi particle description of near extremal branes [5, 3, 4] the thermodynamics can be approximated by free quasi particles. The number of quasi particles is the entropy, the energy of each quasi particle is around the temperature and the life time of the quasi particles is set by the inverse temperature of the field theory.

Now when we have a system of branes and anti-branes we have assumed that at the stability point the theories on them are decoupled from each other except for the constraint that the excitation energy be the same. This is the assumption underlying the derivation of the entropy of the far from extremal black holes. This means that the spectral density of the theories is the same as if the other theory does not exist. From the width of the peaks in the spectral density, or from the scaling of the quasi normal modes in the background of the near extremal branes, one can read off the life time of the quasi particle and is thus proportional to the inverse temperature [4].

Thus we can write (remembering that the number of quasi-particles is proportional to the entropy),

$$\begin{aligned} \frac{dS}{dt} &\sim ST \\ \frac{d\bar{S}}{dt} &\sim \bar{S}\bar{T} \end{aligned} \tag{2.8}$$

where the proportionality constant is the same in both theories. What we mean by theses equations is that the quasi particle decay and their number changes. Off course in equilibrium quasi particles are also created in exactly the same rate, but one can focus on their decay and compare to the corresponding properties of the black hole.

Thus the total entropy $S_{\text{tot}} = S + \bar{S}$ satisfies

$$\frac{dS_{\text{tot}}}{dt} \sim ST + \bar{S}\bar{T} \,. \tag{2.9}$$

However

$$ST + \bar{S}\bar{T} = (S + \bar{S}) \frac{2T\bar{T}}{T + \bar{T}} = S_{\text{tot}} T_{BH} \tag{2.10}$$

where in the last equality we have used that

$$\bar{S}\bar{T} = ST \sim E \,. \tag{2.11}$$

We thus see that the equation (2.9) reproduces equation (2.7).

While the life time of the quasi particle is the same as in the near extremal case, the relationship between the change in the number of quasi particles and the change in

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excitation energy is not the same since pairs of branes and anti branes can be created or annihilated. So while we know the rate of quasi particle decay this does not tell us the rate in which the total mass of the black hole changes. Now in the gravity picture we imagine that the black hole would slowly decay (if there was no incoming flux at the stretched horizon to balance off the decay) and just become a slightly less massive black hole hence the use of (at constant charge Q)

$$dM_{BH} = T_{BH} dS_{BH}\,. \tag{2.12}$$

On the field theory side the brane anti brane system is only a black hole if the energy and the number of brane and anti branes obey the correct relationship (2.3). If we now allow changes in both the excitation energy and number of branes, are we guaranteed to obey (2.12). In particular how restrictive does the fluctuations in energy and brane number need to be.

To try and answer this question let us look at a general change in the energy and brane number in the field theory, using equations (2.2) and (2.3), we can write

$$dS = \frac{\partial S}{\partial E} dE + \frac{\partial S}{\partial M_{p,0}} dM_{p,0} = \frac{1}{T} dE + \frac{1}{\bar{T}} dM_{p,0} \tag{2.13}$$

and similarly

$$d\bar{S} = \frac{1}{\bar{T}} d\bar{E} + \frac{1}{T} dM_{\bar{p},0}\,. \tag{2.14}$$

Now if we look for perturbation that leaves the charge constant then $dM_{\bar{p},0} = dM_{p,0} = dM$, so we can write two equations for the time dependence of the quantities above

$$\begin{aligned} \frac{dS}{dt} &= \frac{1}{T}\frac{dE}{dt} + \frac{1}{\bar{T}}\frac{dM}{dt} \\ \frac{d\bar{S}}{dt} &= \frac{1}{\bar{T}}\frac{d\bar{E}}{dt} + \frac{1}{T}\frac{dM}{dt}\,. \end{aligned} \tag{2.15}$$

If we want to require that the change in the field theory correspond to infinitesimal changes in a black hole configuration then one should have (using equation (2.15))

$$\frac{1}{T}\frac{dE}{dt} + \frac{1}{\bar{T}}\frac{d\bar{E}}{dt} = \frac{1}{T_{BH}}\frac{dE_{\rm tot}}{dt}\,. \tag{2.16}$$

Which is less restrictive then one would have thought, since it does not couple dM to dE.

Now while we have $E = \bar{E}$ for stability one does not have to assume that $\frac{dE}{dt} = \frac{d\bar{E}}{dt}$, since for each there is also a corresponding inflow of energy at equilibrium.[1] This inflow comes form the annihilation of the branes and anti-branes which is converted into excitation energy just as energy excitation creates brane anti brane pairs. Since at equilibrium the number of branes and anti branes stays the same and the energy E is constant this means that

$$\frac{dE}{dt} + \frac{d\bar{E}}{dt} = 2\frac{dM}{dt}\,. \tag{2.17}$$

[1]Even though this might be true in a particular example as above.

We seem to have more equations then unknown.

Solving equations (2.15), (2.16) and (2.17) and we find

$$\frac{dE}{dt} = \frac{d\bar{E}}{dt} = \frac{dM}{dt} \sim ET_{BH}\,. \tag{2.18}$$

From which we can find

$$2\frac{dE + dM}{dt} = \frac{dM_{BH}}{dt} = T_{BH}\frac{dS_{\rm tot}}{dt} \sim S_{\rm tot}T_{BH}^2\,, \tag{2.19}$$

in agreement with equation (2.6).

Having a solution to an over constrained system suggest that the extra condition (2.17) which reflects the requirement of stability of the system is somehow already incorporated in the description. This suggest that the requirement that the energy on brane and anti branes be the same from which the black hole configuration arose is indeed related to stability [11].

It is worth presenting the decay rate for the branes in another form

$$\frac{dM}{dt} \sim \frac{1}{a}\frac{(M\bar{M})^{\frac{3d-2}{4d}}}{\sqrt{M} + \sqrt{\bar{M}}} \tag{2.20}$$

which gives the decay rate as a function of the number of branes and anti branes alone.

3. Multi charged BH

We now look at a more general case involving many types of branes and anti-branes. The mass and entropy are given by [16, 17],

$$\begin{aligned}
M_{bh} &= \frac{b}{2}\mu^{D-3}\left(\sum_{i=1}^{n}\cosh\gamma_i + 2\lambda\right)\\
b &= \frac{\omega_{D-2}}{2\kappa^2}(D-3)V_p\,, \qquad \lambda = \frac{D-2}{D-3} - \frac{n}{2}\\
S_{bh} &= c\mu^{D-2}\Pi_{i=1}^{n}\cosh\gamma_i\\
c &= \frac{4\pi b}{D-3}\\
T_{BH} &= \frac{d}{4\pi\mu\Pi_{i=1}^{n}\cosh\gamma_i}\,.
\end{aligned} \tag{3.1}$$

The field theory on the branes anti-branes configuration can be summarized by [11],

$$\begin{aligned}
M_f &= \sum_{i}^{n}(M_{p_i,0} + M_{\bar{p}_i,0}) + E_{\rm tot}\\
S_f &= \tilde{a}\left(\frac{E_{\rm tot}}{2^n}\right)^{\lambda}\Pi_{i=1}^{N}\left(\sqrt{M_{p_i,0}} + \sqrt{M_{\bar{p}_i,0}}\right)\\
\tilde{a} &= c(b\lambda)^{-\lambda}b^{-n/2}\,.
\end{aligned} \tag{3.2}$$

Where each of the 2^n configuration of branes and anti-branes has the same energy of excitations, but different temperatures. The different temperatures are related to the black hole temperature by

$$\frac{2^n}{T_{BH}} = \sum_{i=1}^{2^n} \frac{1}{T_i} \,. \tag{3.3}$$

Maximizing the entropy for fixed charge and mass gives the relationship

$$E_{\rm tot} = 4\lambda\sqrt{M_{p_i,0}M_{\bar{p}_i,0}}, \tag{3.4}$$

for all i.

In terms of μ and γ one has,

$$\begin{aligned} M_{p_i,0} &= \frac{b}{4}\mu^{D-3}e^{2\gamma_i} \\ M_{\bar{p}_i,0} &= \frac{b}{4}\mu^{D-3}e^{-2\gamma_i} \\ E_{\rm tot} &= b\lambda\mu^{D-3}\,. \end{aligned} \tag{3.5}$$

We now want to recover (2.7) and (2.6) from a quasi-particle description.

As before we assume each of the 2^n field theories is described by quasi-particles with energy of order the temperature and life time of order the inverse temperature.

$$\frac{dS_i}{dt} \sim S_i T_i \qquad i = 1\cdots 2^n\,. \tag{3.6}$$

The time derivative of the total entropy is then

$$\frac{dS_{\rm tot}}{dt} \sim \sum_i S_i T_i \sim S_{\rm tot} T_{BH} \tag{3.7}$$

where in the last equality we have used equation (3.3) and that all the S_iT_i are equal for all i, due to the equality of the energy of excitation on all 2^n brane configurations.

We now want to compute the individual rates of the branes decay and reproduce equation (2.6). When the number of quasi particles changes this can come about by either a change in the energy or a change in the number of branes. But each of the 2^n field theories live on some of the same branes thus they affect each other. While they can not affect the life time of the quasi-particles they can affect the distribution of the available energy between branes and excitations. To be concrete let us look at the example of $n = 2$

3.1 The $n = 2$ case

In this case we have for the entropy

$$S_{\rm tot} = aE^\lambda\left(\sqrt{M_1}\sqrt{\bar{M}_2} + \sqrt{M_1}\sqrt{M_2} + \sqrt{\bar{M}_1}\sqrt{M_2} + \sqrt{\bar{M}_1}\sqrt{\bar{M}_2}\right) \tag{3.8}$$

and we can write it as

$$S_{\rm tot} = S_1 + S_2 + S_3 + S_4\,, \tag{3.9}$$

respectively. The equations for the rate of change are

$$\begin{aligned}
\dot{S}_1 &= \frac{1}{T_1}\dot{E}_1 + \frac{1}{2T_4}\dot{M}_1 + \frac{1}{2T_2}\dot{M}_2 \\
\dot{S}_2 &= \frac{1}{T_2}\dot{E}_2 + \frac{1}{2T_3}\dot{M}_1 + \frac{1}{2T_1}\dot{M}_2 \\
\dot{S}_3 &= \frac{1}{T_3}\dot{E}_3 + \frac{1}{2T_2}\dot{M}_1 + \frac{1}{2T_4}\dot{M}_2 \\
\dot{S}_4 &= \frac{1}{T_4}\dot{E}_4 + \frac{1}{2T_1}\dot{M}_1 + \frac{1}{2T_3}\dot{M}_2 \,.
\end{aligned} \tag{3.10}$$

Where we have taken into account the effect of the field theories on each other by equating the decay rate of M_i on each field theory that live on these branes. Now we have eight unknown but only four equations. Four more equation are given by the equality of the rate of energy decay to brane decay, charge conservation, and the analog of equation (2.16) i.e

$$\sum_i \left(\frac{\dot{E}_i}{T_i}\right) = \frac{1}{T_{BH}}\dot{E}_{\rm tot} \,. \tag{3.11}$$

These together give eight equations which can be solved to give the individual decay rates.

The charge conservation equation plus (3.11) and the structure of (3.10) give

$$dS_{\rm tot} = \frac{1}{T_{BH}}(dE_{\rm tot} + 2dM_1 + 2dM_2) \tag{3.12}$$

ensuring equation (2.6).

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